

# Baryon acoustic oscillations with the cross-correlation of spectroscopic and photometric samples

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## ABSTRACT

The baryon acoustic oscillation (BAO) experiment requires a sufficiently dense sampling of large-scale structure tracers with spectroscopic redshift, which is observationally expensive especially at high redshifts  $z \gtrsim 1$ . Here we present an alternative route of the BAO analysis that uses the cross-correlation of sparse spectroscopic tracers with a much denser photometric sample, where the spectroscopic tracers can be quasars or bright, rare galaxies that are easier to access spectroscopically. We show that measurements of the cross-correlation as a function of the transverse comoving separation rather than the angular separation avoid a smearing of the BAO feature without mixing the different scales at different redshifts in the projection, even for a wide redshift slice  $\Delta z \simeq 1$ . The bias, scatter, and catastrophic redshift errors of the photometric sample affect only the overall normalization of the cross-correlation which can be marginalized over when constraining the angular diameter distance. As a specific example, we forecast an expected accuracy of the BAO geometrical test via the cross-correlation of the SDSS and BOSS spectroscopic quasar sample with a dense photometric galaxy sample that is assumed to have a full overlap with the SDSS/BOSS survey region. We show that this cross-correlation BAO analysis allows us to measure the angular diameter distances to a fractional accuracy of about 10% at each redshift bin over  $1 \lesssim z \lesssim 3$ , if the photometric redshift errors of the galaxies,  $\sigma_z/(1+z)$ , are better than 10–20% level.

**Key words:** cosmology: distance scale – large-scale structure of the Universe

## 1 INTRODUCTION

Various cosmological data sets such as the cosmic microwave background (CMB; Hinshaw et al. 2012), the type-Ia supernova observations (Riess et al. 1998; Schmidt et al. 1998; Perlmutter et al. 1999; Kessler et al. 2009; Suzuki et al. 2012) and the baryon acoustic oscillation (BAO) experiments (Eisenstein et al. 2005; Percival et al. 2007, 2010; Blake et al. 2011; Beutler et al. 2011; Anderson et al. 2012) have shown increasing evidence that the cosmic expansion today is in the accelerating expansion phase. The cosmic acceleration is the most tantalizing problem in cosmology.

Among others, the BAO experiment is recognized as one of the most promising geometrical tests, because it rests on the physics of the CMB anisotropies in the early universe, which is remarkably well described by the linearized perturbation theory for a  $\Lambda$ -dominated cold dark matter model ( $\Lambda$ CDM). The tight coupling between baryons and photons prior to the decoupling epoch of  $z \simeq 1100$  leaves a characteristic imprint on the pattern of large-scale structure tracers such as galaxies and quasars – the so-called BAO scale. The

BAO scale is now precisely constrained as  $\simeq 150$  Mpc from the CMB observations (Hinshaw et al. 2012), which can be used as a “standard ruler” to infer the cosmological distances from the observed correlation function of the tracers (Seo & Eisenstein 2003; Hu & Haiman 2003).

The BAO measurements mostly utilize a large data from wide-area redshift surveys of galaxies, such as the 6dFGS<sup>1</sup>, the Sloan Digital Sky Survey (SDSS)<sup>2</sup>, the Baryon Oscillation Spectroscopic Survey (BOSS)<sup>3</sup>, and the WiggleZ survey<sup>4</sup>. With the success of these surveys, there are several future BAO surveys targeting higher redshift ranges of  $z \gtrsim 1$ , including the HETDEX survey<sup>5</sup>, the Extended Baryon Oscillation Spectroscopic Survey (eBOSS)<sup>6</sup>, the Big-

<sup>1</sup> <http://www.aao.gov.au/6dFGS/>

<sup>2</sup> <http://www.sdss.org/>

<sup>3</sup> <http://www.sdss3.org/surveys/boos.php>

<sup>4</sup> <http://wigglez.swin.edu.au/site/>

<sup>5</sup> <http://hetdex.org>

<sup>6</sup> <http://www.sdss3.org/future/eboos.php>

BOSS<sup>7</sup>, the Subaru Prime Focus Spectrograph project<sup>8</sup> (Ellis et al. 2012), and the satellite Euclid mission<sup>9</sup>. However, extending the BAO experiment to higher redshifts ( $z \sim 2 - 4$ ) is observationally expensive, because the target galaxies become increasingly fainter and spectroscopic surveys of such faint galaxies covering wide-areas are quite time-consuming<sup>10</sup>.

In addition to these spectroscopic BAO analysis, there have been attempts to measure the BAO feature in the correlation function of photometric galaxy samples (Padmanabhan et al. 2007; Blake et al. 2007; Carnero et al. 2012; Seo et al. 2012). A wide-area, multi-colour photometric survey is relatively easy to carry out compared to a spectroscopic survey of similar area coverage. In fact, there are many planned imaging surveys, including the Subaru Hyper Suprime-Cam (HSC) Survey<sup>11</sup>, the Dark Energy Survey (DES)<sup>12</sup>, Euclid, and the LSST project<sup>13</sup>, for which the primary science driver is weak lensing based cosmology. However, the photometric BAO experiments are challenging for several reasons. First, the photometric BAO analysis is based on the angular correlation function of the galaxies, which is by nature two-dimensional and loses (wash out) the clustering information in the line-of-sight direction that also causes a reduction in the correlation function amplitude. Second, the projection along the line-of-sight smears the BAO feature in the angular power spectrum. Third, the BAO feature inferred from the photometric samples can be significantly affected by statistical and systematic (catastrophic) errors of the photometric redshifts (photo- $z$ 's). For example, including the photo- $z$  outliers in the analysis can easily induce a bias in the BAO peaks, which in turn causes a bias in the inferred distance.

In this paper, we propose to use the cross-correlation between the spectroscopic and photometric tracers of large-scale structure as an alternative BAO method. This method is particularly useful when sampling of the spectroscopic tracers is too sparse to measure the BAO feature via its auto-correlation analysis. Since a photometric survey usually has a much denser sampling, the cross-correlation mitigates the shot noise contamination to improve clustering measurements. We argue that smearing due to the line-of-sight projection can be avoided by measuring the correlation function as a function of the transverse comoving separation rather than the angular separation. As a specific example, we consider the cross-correlation of the SDSS and BOSS spectroscopic sample of quasars with photometric galaxies to estimate an expected accuracy of the derivable geometrical test. The SDSS quasars are bright and can easily be observed spectroscopically, but have a too sparse sampling for the auto-correlation analysis. When making the forecast, we also include the broad-band shape information of the

cross-correlation in addition to the BAO feature (also see Cooray et al. 2001).

This paper is organized as follows. In Section 2, we describe explicit expressions for the cross-correlation analysis as a function of the transverse comoving separation as well as its counterpart in Fourier space, and also derive the covariance matrix. We show our basic results in Section 3. In Section 4, we show an expected accuracy of the geometrical test via the use of the cross-correlation of the SDSS/BOSS spectroscopic quasar sample with a dense photometric galaxy sample. We summarize our results in Section 5. Unless otherwise stated, we employ a concordance  $\Lambda$ CDM model, with  $\Omega_{m0}h^2 = 0.137$  and  $\Omega_b h^2 = 0.023$  for the matter and baryon physical density parameters,  $\Omega_\Lambda = 0.721$  for the cosmological constant assuming a flat geometry, and  $A_s = 2.43 \times 10^{-9}$ ,  $n_s = 0.96$ , and  $\alpha_s = 0$  for the primordial power spectrum parameters.

## 2 BAO FEATURE IN THE PROJECTED CORRELATION FUNCTION

### 2.1 Transverse cross-correlation function and the power spectrum

In this paper, we consider a method using the cross-correlation of a photometric sample with a spectroscopic sample for measuring the BAO scale. A key idea is to consider the cross-correlation measured as a function of the transverse comoving separation rather than of an angular separation

$$w(R) \equiv \frac{1}{\bar{n}_s \bar{n}_p} [\langle n_s(\boldsymbol{\gamma}_s; z_s) n_p(\boldsymbol{\gamma}_p) \rangle - 1], \quad (1)$$

where quantities with subscripts “s” and “p” denote those for spectroscopic and photometric samples, respectively;  $n_s(\boldsymbol{\gamma}_s; z_s)$  and  $n_p(\boldsymbol{\gamma}_p)$  are the projected number density fields for the spectroscopic and photometric samples in the directions of  $\boldsymbol{\gamma}_s$  and  $\boldsymbol{\gamma}_p$  on the celestial sphere, respectively;  $z_s$  is the redshift of each spectroscopic object. Thus the density field of the spectroscopic sample is described as a function of both  $z_s$  and the angular position. The transverse radius  $R$  is defined in terms of their observed angular positions of  $\boldsymbol{\gamma}_s$  and  $\boldsymbol{\gamma}_p$  and the redshift  $z_s$  as

$$R = d_A(z_s) \cos^{-1}(\boldsymbol{\gamma}_s \cdot \boldsymbol{\gamma}_p) \simeq d_A(z_s) |\boldsymbol{\theta}_s - \boldsymbol{\theta}_p|. \quad (2)$$

The quantity  $d_A(z_s)$  is the comoving angular diameter distance to each spectroscopic object. Note that this conversion requires to assume a background cosmological model. The unit vector on the celestial sphere,  $\boldsymbol{\gamma}$ , is given as  $\boldsymbol{\gamma} \equiv (\sin \vartheta \cos \varphi, \sin \vartheta \sin \varphi, \cos \vartheta)$ . In the last equality on the r.h.s. of Eq. (2), we used the flat-sky approximation<sup>14</sup>. Observationally, the cross-correlation is estimated from the average of all the pairs separated by the same separation  $R$  within a given width, compared to the cross-correlation of the spectroscopic sample with random catalogues that are

<sup>7</sup> <http://bigboss.lbl.gov/>

<sup>8</sup> <http://sumire.ipmu.jp/en/2652>

<sup>9</sup> <http://sci.esa.int/science-e/www/object/index.cfm?fobjectid=48983>

<sup>10</sup> We note that the BAO feature at  $z \lesssim 2.3$  is recently detected from the three-dimensional correlation function of the Lyman- $\alpha$  forests that are identified in the BOSS quasar spectra (Busca et al. 2012; Slosar et al. 2013).

<sup>11</sup> <http://www.naoj.org/Projects/HSC/index.html>

<sup>12</sup> <http://www.darkenergysurvey.org>

<sup>13</sup> <http://www.lsst.org/lsst/>

<sup>14</sup> In the flat-sky approximation, the unit vector is expanded around the North pole as  $\boldsymbol{\gamma} \simeq (\vartheta \cos \varphi, \vartheta \sin \varphi, 1)$ , and the two-dimensional flat-space vector can be defined as  $\boldsymbol{\theta} \equiv (\vartheta \cos \varphi, \vartheta \sin \varphi)$ .

constructed based on the same selection function as in the photometric catalogue.

A notable advantage of the  $R$ -average over the angle average is that it can preserve the physical scales inherent in large-scale structure such as the scale of baryon acoustic oscillation in which we are interested, *if* the assumed cosmological model for the  $R$ - $\theta$  conversion is closer to the true underlying cosmology. On the other hand, the  $\theta$ -average mixes different scales in large-scale structure, thus smearing the BAO scale in the observed cross-correlation function. We emphasize that this  $R$ -average is useful when a spectroscopic catalogue is available for the cross-correlation measurement. In contrast, in the case of the auto-correlation analysis of photometric samples, the conversion from  $\theta$  to  $R$  is severely affected by photo- $z$  uncertainties, which can lead to a smearing or systematic offset of the BAO feature.

We can express the projected cross-correlation function in terms of the power spectrum as follows. First, in the case that the spectroscopic sample has a redshift distribution, the projected cross-correlation can be expressed as

$$w(R) = \int_0^\infty dz_s p_s(z_s) \tilde{w}(\theta; z_s)|_{R=d_A(z_s)\theta}, \quad (3)$$

where  $\tilde{w}(\theta; z_s)$  is the angular cross-correlation function between a spectroscopic sample at redshift  $z_s$ ,  $p_s(z_s)$  is the redshift distribution of the spectroscopic sample, normalized as  $\int_0^\infty dz_s p_s(z_s) = 1$ , and the average with the notation  $|_{R=d_A(z_s)\theta}$  indicates that the redshift average for a given  $R$  is done by averaging the angular correlation function  $\tilde{w}(\theta; z_s)$  under the condition  $R = d_A(z_s)\theta$  according to the discussion around Eqs. (1) and (2).  $\tilde{w}(\theta; z_s)$  is defined in terms of the angular power spectrum as

$$\tilde{w}(\theta; z_s) = \frac{1}{4\pi} \sum_l (2l+1) C_{sp}(l; z_s) P_l(\cos \theta). \quad (4)$$

Here  $P_l(x)$  is the  $l$ -th order Legendre polynomials. For a flat universe, the angular power spectrum is given in terms of the three-dimensional power spectrum, in a standard manner (e.g., Dodelson 2003), as

$$C_{sp}(l; z_s) \equiv \frac{2}{\pi} \int dr W_p(r) \int k^2 dk P_{sp}(k; z_s, z) j_l(kr_s) j_l(kr), \quad (5)$$

where  $P_{sp}(k; z_s, z)$  is the three-dimensional cross-power spectrum between the spectroscopic objects at redshift  $z_s$  and photometric objects at  $z$ ;  $r$  is the radial distance given as a function of redshift for a given cosmology,  $r = r(z)$ , and  $r_s = r(z_s)$ ;  $j_l(x)$  is the  $l$ -th order spherical Bessel function;  $W_p(r)$  is the selection function of the photometric sample, normalized as  $\int dr W_p(r) = 1$  (see below for an example). Note that  $r(z) = d_A(z)$  for a flat universe.

Using the flat-sky approximation and the Limber's approximation (Limber 1954), the projected cross-correlation function can be simplified as

$$w(R) \equiv \int \frac{k dk}{2\pi} C_{sp}(k) J_0(kR), \quad (6)$$

where  $J_0(x)$  is the zeroth-order Bessel function, and the transverse comoving separation  $R$  is defined for spectroscopic redshift  $z_s$  of each sample used in the average. The projected cross-power spectrum  $C_{sp}(k)$  is given by a

simple form:

$$C_{sp}(k) \equiv \int dr p_s(z) \frac{dz}{dr} W_p(r) P_{sp}(k; z). \quad (7)$$

Note that the power spectrum  $C_{sp}(k)$  has a dimension of  $\text{Mpc}^2$  so that  $k^2 C_{sp}(k)$  becomes dimension-less. We use  $C_{sp}(l)$  for the usual angular power spectrum and  $C_{sp}(k)$  for the Fourier counterpart of the  $w(R)$  throughout the paper.

We assume that we can, based on photo- $z$  technique, select photometric objects that have similar photo- $z$  to the spectroscopic redshift. Even in the presence of large photo- $z$  errors, the cross-correlation method is very powerful in the sense that it can statistically select photometric objects that are physically clustering with the spectroscopic sample (Newman 2008; McQuinn & White 2013). Including photo- $z$  bias and outliers in the sample simply dilutes the cross-correlation signals, but does not change the shape so that the BAO scale is not shifted. If the spectroscopic sample used in the cross-correlation measurement is in a narrow range of redshifts,  $[z_s, z_s + \Delta z_s]$ , the projected power spectrum reads

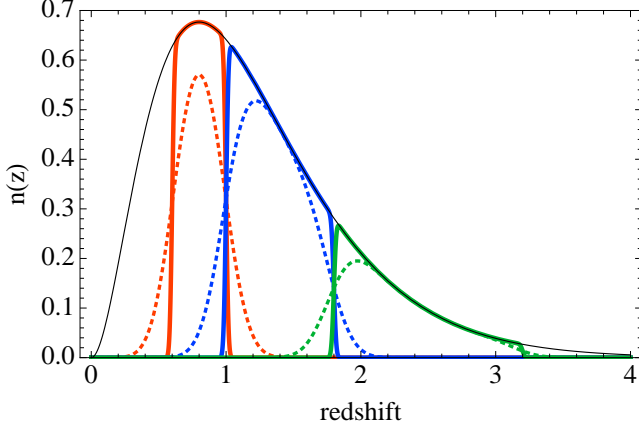
$$C_{sp}(k) \simeq \left[ \int_{r_s}^{r_s + \Delta r_s} dr W_p(r) \right] \frac{1}{\Delta r_s} P_{sp}(k; z_s), \quad (8)$$

where  $r_s$  and  $r_s + \Delta r_s$  are the radial distances to redshifts  $z_s$  and  $z_s + \Delta z_s$ , respectively, and we have used  $\Delta r_s p_s(z_s)(dz/dr) = 1$ . The factor  $\int_{r_s}^{r_s + \Delta r_s} dr W_p(r)$  is a fraction of photometric objects among the whole photometric sample that reside in the spectroscopic redshift bin  $[z_s, z_s + \Delta z_s]$  and thus are physically correlated with the spectroscopic sample. Therefore the factor gives a dilution factor of the cross-correlation signal due to the photo- $z$  errors. The factor  $1/\Delta z_s$  in front of  $P_{sp}(k)$  accounts for the fact that the cross-correlation amplitude is reduced with increasing the width of spectroscopic redshift bin. The above equation explicitly shows that an inclusion of the photo- $z$  outliers does not change the shape of the cross-correlation, but simply affects the overall normalization. Also importantly, the projected cross-correlation, measured against  $R$  instead of the angular separation, can measure the three-dimensional power spectrum  $P_{sp}(k; z_s)$  at spectroscopic redshift and at a particular  $k$ . Put another way, the projected cross-correlation does not mix the power spectrum of different Fourier modes, which is not the case for the angular power spectrum. It should also be noted that the projected cross-correlation is not affected by redshift-space distortion due to the peculiar motions of the tracers. In particular, the nonlinear redshift-space distortion, the so-called Finger-of-God effect, is very difficult to accurately model (Hikage et al. 2012), and therefore the cross-correlation may have a practical advantage.

Next let us consider a case that the spectroscopic sample is in a given width of the redshift bin,  $z_s = [z_s^{\text{low}}, z_s^{\text{up}}]$ , where  $z_s^{\text{low}}$  and  $z_s^{\text{up}}$  are the lower and upper bound of the redshift bin respectively. Here for simplicity we consider a uniform distribution of the spectroscopic sample within the given redshift bin

$$p_s(z) = \begin{cases} \frac{1}{\Delta z_s} & \text{if } z \in [z_s^{\text{low}}, z_s^{\text{up}}], \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

where  $\Delta z_s \equiv z_s^{\text{up}} - z_s^{\text{low}}$ . The uniform redshift distribution is not a critical assumption, and can be easily generalized to



**Figure 1.** The thin solid line shows a toy model of a photometric galaxy distribution. Among them, we select galaxies which fall in the spectroscopic redshift ( $z_s$ ) bin. The thick solid lines show the underlying true redshift distribution of the photometric samples which are defined by the photo- $z$  bins of  $z_p \in [0.6, 1.0]$ ,  $[1.0, 1.8]$ , and  $[1.8, 3.2]$ , assuming a photo- $z$  accuracy of  $\sigma_z = 0.01(1 + \bar{z}_s)$  (see Eq. 10). The dotted lines are similar distributions but the photo- $z$  errors are degraded to  $\sigma_z = 0.1(1 + \bar{z}_s)$ .

a case that the spectroscopic sample has a redshift distribution. For photometric objects used for the cross-correlation measurement, we would select the objects if the best-fit photo- $z$ 's are in the range of the spectroscopic redshift bin. Here we employ the simplified assumption that the probability for photo- $z$ 's obeys a Gaussian distribution

$$p_p(z|z_p) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left[-\frac{(z_p - z)^2}{2\sigma_z^2}\right], \quad (10)$$

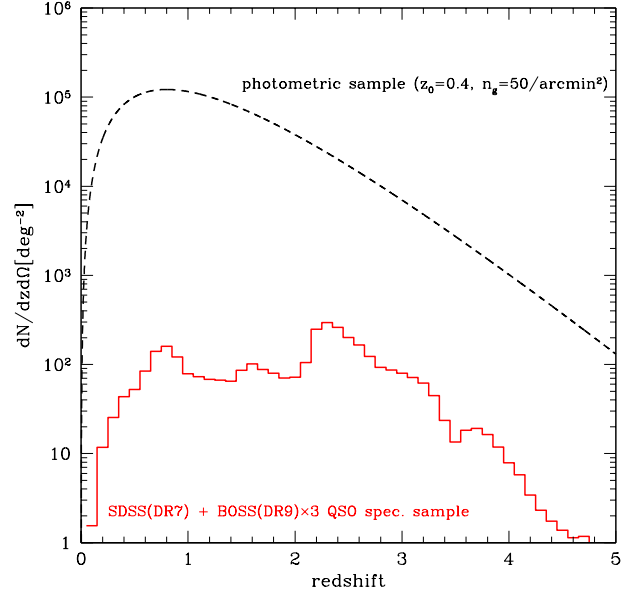
where we assumed the photometric objects with the best-fit photo- $z$ ,  $z_p$ , obeys a single population,  $\sigma_z$  is the  $1\sigma$  photo- $z$  error, and  $z$  is its true redshift. The probability satisfies the normalization  $\int_{-\infty}^{\infty} dz_p p_p(z|z_p) = \int_{-\infty}^{\infty} dz p_p(z|z_p) = 1$ . Given the distribution, if the photometric objects whose photo- $z$ 's are in the range of the spectroscopic redshift range,  $z_p \in [z_s^{\text{low}}, z_s^{\text{up}}]$ , the probability distribution for the true redshift is computed as

$$\begin{aligned} p_p(z|z_p \in [z_s^{\text{low}}, z_s^{\text{up}}]) &= \int_{z_s^{\text{low}}}^{z_s^{\text{up}}} dz_p p_p(z|z_p) \\ &= \frac{1}{2} \left[ \text{erf}(x^{\text{up}}) - \text{erf}(x^{\text{low}}) \right], \end{aligned} \quad (11)$$

where  $\text{erf}(x)$  is the error function and  $x^{\text{up/low}} = (z_s^{\text{up/low}} - z)/\sqrt{2}\sigma_z$ . Taking into account the overall redshift distribution of the photometric sample, we can derive the redshift distribution of the photometric sample based on the photo- $z$  selection:

$$n_{p \in z_s}(z) = \frac{1}{2} n_p(z) \left[ \text{erf}(x^{\text{up}}) - \text{erf}(x^{\text{low}}) \right], \quad (12)$$

where  $n_p(z)$  is the redshift distribution of the photometric sample for which we assume  $n_p(z) = (z^2/2z_0^3) \exp[-z/z_0]$  parametrized by  $z_0$  (see Oguri & Takada 2011). Throughout the paper we assume  $z_0 = 0.4$ , yielding the mean redshift  $\langle z \rangle = 1.2$ , and assume the bias parameter of the photometric sample to  $b_p = 1.5$ . Hence, the selection function of the photometric sample in each  $z_s$  bin used for the cross-correlation



**Figure 2.** The histogram shows the redshift distribution of the spectroscopic quasar sample of SDSS DR7 (Schneider et al. 2010) and BOSS DR9 (Paris et al. 2012). The BOSS sample is multiplied by 3 as the DR9 has completed only the 1/3 of the target area. Dashed line shows the assumed photometric galaxy distribution,  $n_p(z)$  defined below Eq. (12), where we assumed  $\bar{n}_p = 50 \text{ arcmin}^{-2}$  for the total mean number density and  $\langle z \rangle = 1.2$  for the mean redshift.

is given as

$$W_p(r) = \frac{n_{p \in z_s}(z)}{\bar{n}_{p \in z_s}} \frac{dz}{dr}, \quad (13)$$

where  $\bar{n}_{p \in z_s}$  is the normalization factor, defined as  $\bar{n}_{p \in z_s} \equiv \int_0^\infty dz n_{p \in z_s}(z)$ , so as to satisfy the condition  $\int_0^\infty dr W_p(r) = 1$ . We will study how an accuracy of the BAO measurement changes with the quantities such as  $\sigma_z$  and  $\Delta z_s$  as well as the number densities of spectroscopic and photometric samples.

In Fig. 1, we show an example of the photometric galaxy distributions with different photo- $z$  uncertainties, when the photometric galaxies are divided in a redshift bins, which is chosen to match with the spectroscopic sample. In this example, we divide the whole sample into three subsamples as  $z_p \in [0.6, 1.0]$ ,  $[1.0, 1.8]$ ,  $[1.8, 3.2]$ . The photo- $z$  errors cause a leakage of the photometric galaxies from the spectroscopic redshift bin.

## 2.2 Covariance matrix

The error covariance matrix quantifies an accuracy of measuring the projected cross-correlation for a given survey. Since angular scales at different redshifts are scaled to match the transverse comoving scale for an assumed cosmological model, the measured projected cross-correlation is two-dimensional, given as a function of the comoving scales in units of Mpc. Assuming a Gaussian error for the projected power spectrum, which is a good approximation at BAO scales (Takahashi et al. 2009), we can extend the standard

formula for the covariance matrix for angular power spectra (Knox 1995) to obtain the covariance matrix of the projected cross-correlation function as

$$\text{Cov}[C_{sp}(k), C_{sp}(k')] = \frac{\delta_{kk'}^K}{N_{\text{mode}}(k)} \times \left[ C_{sp}(k)^2 + \left( C_{ss}(k) + \frac{1}{\bar{n}_s} \right) \left( C_{pp}(k) + \frac{1}{\bar{n}_{p \in z_s}} \right) \right], \quad (14)$$

where  $N_{\text{mode}}(k)$  is the number of independent Fourier mode discriminated by the given survey area defined as

$$N_{\text{mode}}(k) = \frac{2\pi k \Delta k}{\left( \frac{2\pi}{d_A(z_s^{\text{low}}) \Theta_s} \right)^2} = 2k \Delta k d_A(z_s^{\text{low}})^2 f_{\text{sky}}, \quad (15)$$

with  $f_{\text{sky}}$  being the sky coverage defined as  $f_{\text{sky}} \equiv \Omega_s/4\pi$ . The quantities  $\bar{n}_s$  and  $n_{p \in z_s}$  are the projected number densities of the spectroscopic sample and the photometric sample having photo- $z$ 's values within the spectroscopic redshift bin, respectively. The number densities are in units of  $\text{Mpc}^{-2}$ . In the above equations, we assumed that the fundamental model of the two-dimensional Fourier decomposition is defined as the projected scale at the lowest redshift for a given survey area,  $k_f \equiv 2\pi/[d_A(z_s^{\text{low}}) \Theta_s]$ .

The expression for the covariance matrix (Eq. 14) can be used to understand why the cross-correlation can be useful for the BAO analysis when the spectroscopic catalogue has too sparse sampling of the targets, i.e.,  $C_{ss} \ll 1/\bar{n}_s$ . We assume that the photometric sample has a high number density in the spectroscopic redshift bin, i.e.,  $C_{pp} \gg 1/\bar{n}_{p \in z_s}$ , at the BAO scale, and the cross-correlation coefficient between the spectroscopic and photometric samples,  $r \equiv C_{sp}/\sqrt{C_{ss}C_{pp}}$ , is of order unity. In this case, the covariance for the cross-correlation becomes  $\text{Cov}[C_{sp}, C_{sp}] \propto C_{pp}/\bar{n}_s$ , and thus the signal-to-noise ratio is  $(\text{S/N})_{sp}^2 = C_{sp}^2 \text{Cov}^{-1} \propto r^2 C_{ss} \bar{n}_s$ . This should be compared with the case of the auto-power spectra of the spectroscopic sample,  $(\text{S/N})_{ss}^2 \propto (C_{ss} \bar{n}_s)^2$ , yielding the S/N ratio  $(\text{S/N})_{sp}^2/(\text{S/N})_{ss}^2 \simeq r^2/(C_{ss} \bar{n}_s) \gg 1$ . Thus the cross-correlation can give a higher signal-to-noise ratio for a measurement of the projected power spectrum at the BAO scales. In practice, however, the spectroscopic sample allows a measurement of the three-dimensional power spectrum, which contains more Fourier modes than in the projected power spectrum. In the next section, we present a more quantitative comparison between the methods using the cross-correlation and the three-dimensional auto-correlation.

Fig. 2 shows the photometric and spectroscopic samples for which we think the cross-correlation method discussed in this paper is useful, *if* the two survey regions are overlapped. The spectroscopic quasar catalogues of the SDSS and BOSS surveys have a wide coverage of redshift up to  $z_s \simeq 4$ , but have a much lower number density than in the photometric galaxies available from the upcoming imaging surveys such as the Subaru HSC Survey or Euclid. The redshift distribution of the photometric sample shown in Fig. 2 is deeper than what is usually assumed for the HSC survey or Euclid, but we note that for the cross-correlation analysis we can use fainter galaxies than galaxies used for the weak lensing analysis. We also note that our results are not very sensi-

tive to the choice of the number density distribution of the photometric sample.

### 3 RESULT

#### 3.1 Projected power spectra

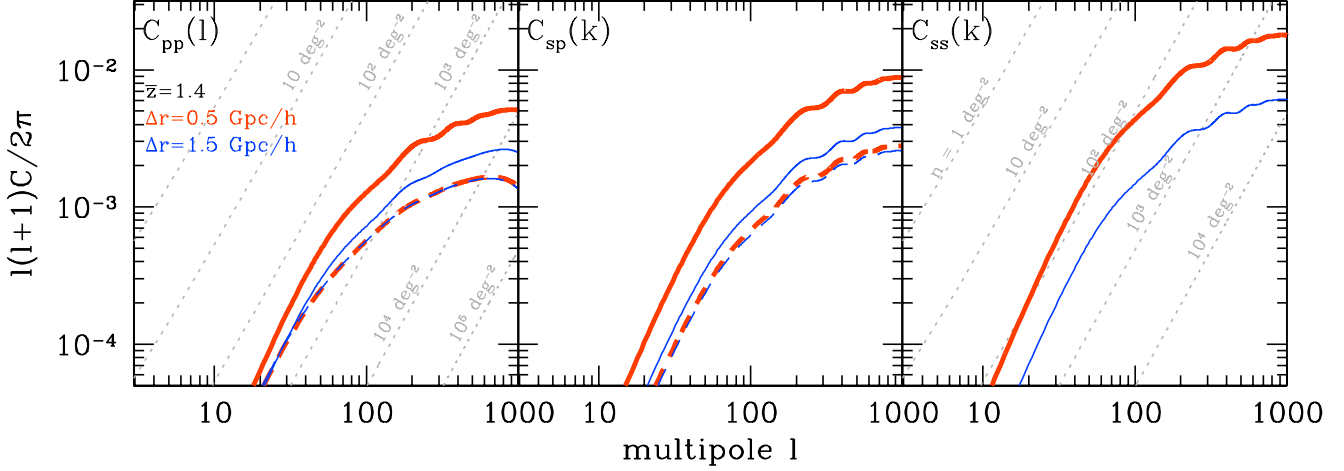
In Fig. 3 we compare auto- and cross-power spectra for spectroscopic and photometric samples at mean redshift  $\bar{z} = 1.4$ . Here the cross-power spectra is computed as a function of the transverse comoving separation as described in the previous section. Here, we consider the redshift bin around  $z = 1.4$ ,  $z = [z - \Delta z/2, z + \Delta z/2]$  with widths  $\Delta z = 0.365$  and 1.05, which correspond to the radial distance widths of  $\Delta r = 0.5$  and 1.5  $\text{Mpc}/h$ , respectively. We assumed that we can select objects in the redshift bin, based on the spectroscopic redshift or photo- $z$ 's. To model the photo- $z$  errors, we use the parametrization given in Ma et al. (2006) as

$$\sigma_z = \lambda_z(1 + \bar{z}_s), \quad (16)$$

and consider the two cases of  $\lambda_z = 0.05$  and 0.3. The BAO feature is smeared for the angular auto-power spectra of photometric samples, while the BAO feature persists in the projected auto- or cross-power spectra using the spectroscopic sample (see also below). Comparing the solid and dashed curves shows that the larger photo- $z$  errors cause a more significant dilution of the power spectrum amplitudes, thereby smearing the BAO oscillatory feature. The figure also shows the shot noise levels. For ongoing or upcoming imaging surveys, we typically have more than  $10^4 \text{deg}^{-2}$  galaxies (see Fig. 2), and thus the power spectrum measurement has a sufficient signal to noise ratio. However, the photo- $z$  errors dilute the spectrum amplitude and smear the BAO feature, suggesting that it would be difficult to use the angular auto-power spectrum of the photometric galaxies for an unbiased BAO geometrical test, as we will discuss below. While the projected cross-power spectra shown in the figure also have a more diluted amplitude with increasing photo- $z$  uncertainties (see Eq. 8), we can still use the unsmeared BAO feature for estimating the angular diameter distance.

#### 3.2 Prospect for the cross-correlation BAO measurement

In this section, we study prospects for the use of the projected power spectrum for measuring the BAO feature. In Fig. 4, we show the projected cross-power spectrum as a function of the transverse wavenumber, divided by the no-wiggle power spectrum (with the BAO feature being smoothed out), in order to highlight the BAO feature. Note that we used the transfer function in Eisenstein & Hu (1998) to compute the no-wiggle spectrum for the same cosmological model. Although we assume a linear bias multiplicative-factor for both the spectroscopic and photometric samples, we include the effect of nonlinear clustering on the matter power spectrum, using the publicly available code, RegPT (Taruya et al. 2012), that includes up to the 2-loop order contributions based on the refined perturbation theory. We show the cross-power spectra up to a certain maximum wavenumber,  $k_{\text{max}}$ , which is determined so that the nonlinear matter power spectrum at the mean redshift is expected



**Figure 3.** Comparison of the projected and angular power spectra at the mean redshift  $\bar{z} = 1.4$ . (*Left*): The angular auto-power spectrum of photometric samples in the photo- $z$  bins  $z_p = [1.4 - \Delta z/2, 1.4 + \Delta z/2]$ . The thick and thin curves show the power spectra for the bin width of  $\Delta z_p \simeq 0.365$  and  $1.05$ , which corresponds to the width of the comoving radial distance,  $\Delta r = 0.5$  and  $1.5$  Gpc/h, respectively. The solid and dashed curves are the spectra assuming the photo- $z$  accuracies of  $\sigma_z/(1+z) = 0.05$  or  $0.3$ , respectively. Each thin dotted lines show the shot noise level for the photometric samples, which typically have the projected number density more than  $10^4 \text{ deg}^{-2}$  for an imaging survey we are interested in. (*Middle*): Similar to the left panel, but for the cross-power spectra between the spectroscopic and photometric samples, as a function of the transverse comoving separation (Eq. 7), where the transverse mode  $k$  is rescaled to the multipole via the distance to the spectroscopic sample by  $l = kr(z = 1.4)$  for an illustrative purpose. The solid and dashed curves are for the photo- $z$  accuracies of the photometric galaxies, as in the left panel. The cross-correlation preserves the BAO wiggles compared to the left panel. (*Right*): The projected auto-power spectra for the spectroscopic samples. The figure shows that, for a spectroscopic survey with a small number density  $\bar{n}_s < 10^2 \text{ deg}^{-2}$ , the BAO wiggles in the auto-spectrum are difficult to measure due to the significant shot noise.

to be accurate to within a 1%-level accuracy in the amplitude compared to the simulation (Taruya et al. 2009, 2012). The figure clearly shows that the projected cross-power spectrum preserves the BAO feature, even for a wide redshift bin of the projection. On the other hand, the BAO feature is smeared in the angular correlation. We also notice that, for a higher redshift slice, the BAO feature remains up to the greater wavenumber due to the less evolving nonlinearities.

We estimate prospects for detecting the BAO feature in the projected cross-spectrum by using the  $\chi^2$ -difference between the power spectra with and without the BAO feature

$$\Delta\chi^2 \equiv \sum_i \frac{[C_{sp}(k_i) - C_{sp}^{nw}(k_i)]^2}{\text{Cov}[C_{sp}(k_i), C_{sp}(k_i)]}, \quad (17)$$

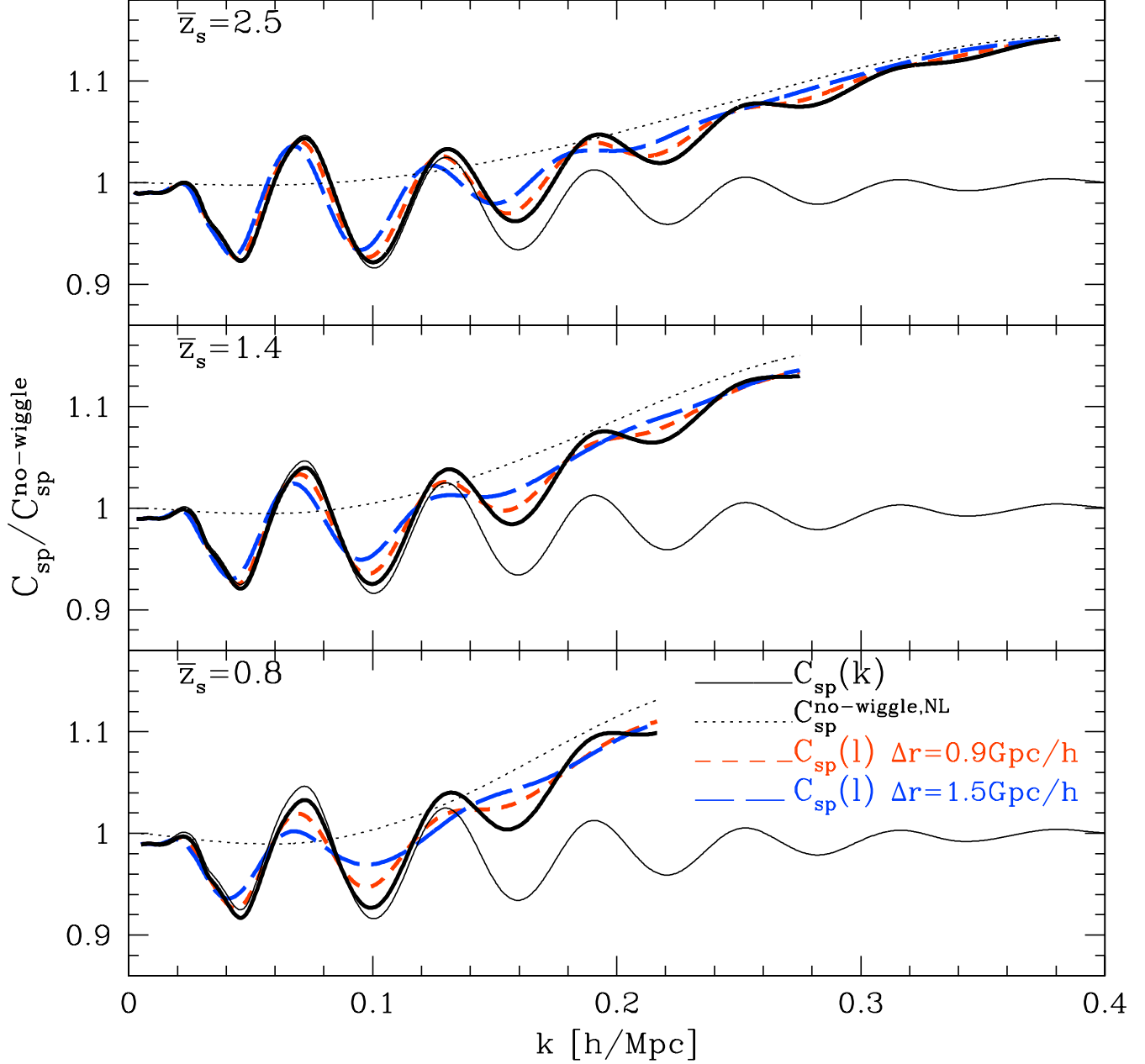
where  $C_{sp}$  and  $C_{sp}^{nw}$  are the cross-power spectra with and without the BAO feature, and the summation is up to the maximum wavenumber determined as in Fig. 4. Note that  $\Delta\chi^2$  does not include the broad-band shape information of the cross-power spectrum, and quantifies a significance of detecting the BAO feature in the cross-power spectrum, assuming that the spectrum with the BAO wiggles is the underlying true spectrum (see the next section for a more quantitative forecast of the BAO analysis). The denominator is the covariance matrix (Eq. 14) for which we assumed the Gaussian error. To compute  $\Delta\chi^2$ , we assume that a spectroscopic sample has the projected number density of  $\bar{n}_s = (20\Delta z) \text{ deg}^{-2}$  in the redshift bin, where  $\Delta z$  denotes the redshift bin width. For the photometric sample, we employ  $\bar{n}_p^{\text{tot}} = 1.8 \times 10^5 \text{ deg}^{-2}$  ( $50 \text{ arcmin}^{-2}$ ) for the *total* number

density. These numbers roughly resemble the SDSS/BOSS quasar spectroscopic sample and the Subaru HSC Survey or Euclid imaging surveys, respectively. We here assume a full-sky coverage ( $f_{\text{sky}} = 1$ ) for both the spectroscopic and photometric catalogues;  $\Delta\chi^2 \propto f_{\text{sky}}$ . We then assume that we can select the photometric objects, which have photo- $z$ 's in the spectroscopic redshift bin, and take into account the redshift distribution of photometric objects as well as the effect of photo- $z$  errors using the method in Sec. 2.1.

Fig. 5 shows the  $\Delta\chi^2$  values for the cross-power spectra assuming a various combination of the survey parameters. If the two surveys have a sufficiently wide area coverage for their overlapping region, the projected cross-power spectrum allows a detection of the BAO feature. We compare the results with a BAO analysis for the spectroscopic sample alone. Similarly to Eq. (17), we can define the differential  $\chi^2$  to quantify the sensitivity of the three-dimensional power spectrum to the BAO feature

$$\Delta\chi_{3D}^2 \equiv \frac{1}{V_{\text{survey}}} \int \frac{2\pi k^2 dk}{(2\pi)^3} \left[ \frac{P_s(k) - P_s^{nw}(k)}{P_s(k) + \bar{n}_s^{-1}} \right]^2, \quad (18)$$

where  $P_s(k)$  is the three-dimensional power spectrum of the spectroscopic sample and  $V_{\text{survey}}$  is the survey volume. We ignore the redshift-space distortion for simplicity (Kaiser 1987). The figure shows that, if the photo- $z$  accuracies of  $\sigma(z_p)/(1+z)$  are better than 10–20%, the cross-correlation can achieve a more significant detection of the BAO feature than in the three-dimensional power spectrum.

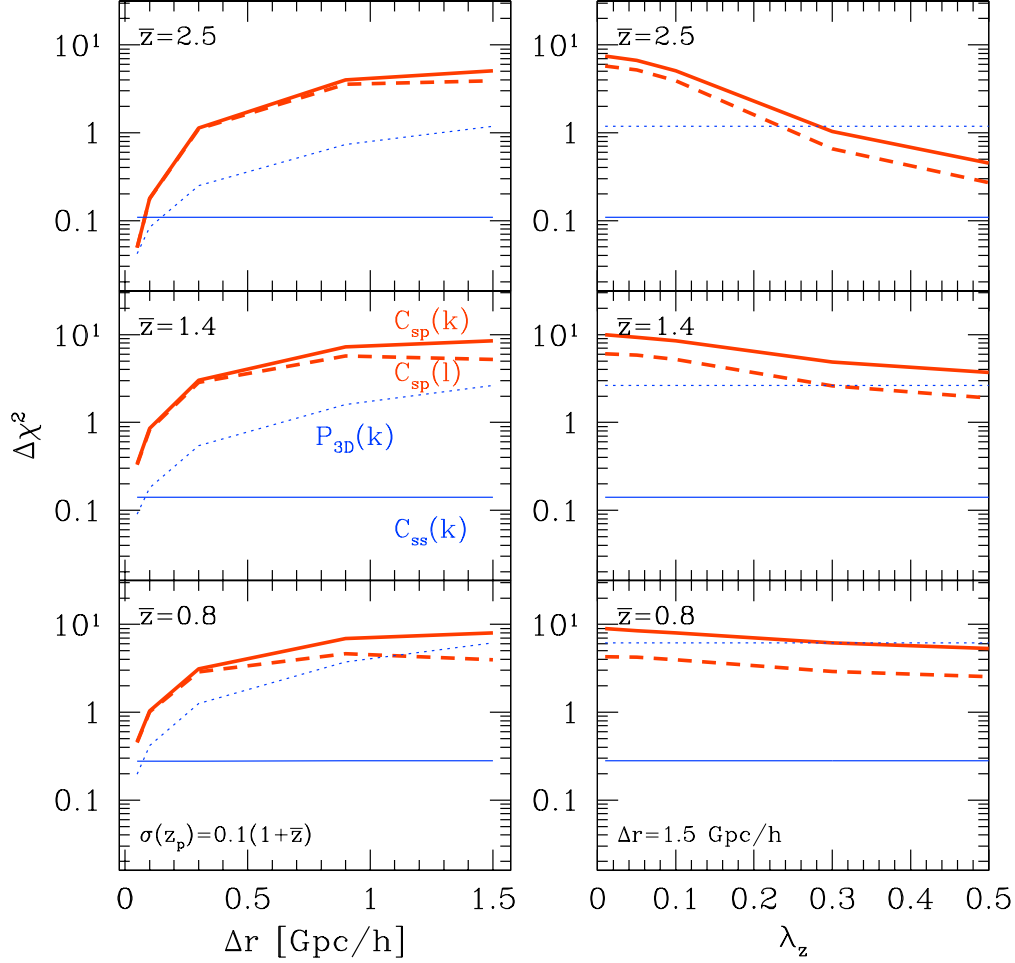


**Figure 4.** The projected power spectrum divided by the no-wiggle linear power spectrum in order to highlight the BAO feature, where we used Eisenstein & Hu (1998) to compute the no-wiggle spectrum. We consider  $\bar{z}_s = 2.5$  (*top panel*), 1.4 (*middle*), and 0.8 (*bottom*), respectively, for the mean redshift of the projection. The thick curves are the spectra computed taking account of the non-linearity of the matter power spectrum (see text for the details), while thin curves show the cross-power spectra in linear theory. Note that we assume the linear bias parameter of  $b = 1$  for both the spectroscopic and photometric samples for simplicity. The spectra are plotted up to  $k_{\text{max}}$ , where the nonlinear power spectra are expected to be accurate to within 1% at each mean redshift compared to simulations. For comparison, we also show the angular cross-power spectra projected over the redshift slice of  $\Delta r = 0.9 \text{ Gpc}/h$  (*short-dashed*) and  $\Delta r = 1.5 \text{ Gpc}/h$  (*long-dashed*) around each mean redshift, respectively, which are plotted against the wavenumber using the conversion  $kr(\bar{z}) = l$ . The dotted curves are the nonlinear power spectrum using the no-wiggle linear power spectrum for the input spectrum.

#### 4 GEOMETRICAL TEST WITH THE CROSS-CORRELATION METHOD

In this section, we present more quantitative estimates on the power of the cross-correlation method for determining the angular diameter distance. For this forecast, in

contrast to the preceding section, we include the broadband shape information of the cross-power spectrum, extending the method in Seo & Eisenstein (2003) to a two-dimensional cross-correlation analysis. As a specific example, here we consider the cross-correlation BAO analysis assuming the SDSS/BOSS spectroscopic quasar catalogues



**Figure 5.** An expected significance of the BAO detection,  $\Delta\chi^2$  (Eq. 17), for the cross-correlation analysis with different combinations of spectroscopic and photometric samples. We estimate the significance by comparing the cross-power spectra with and without the BAO wiggles, as in Fig. 4, but do not include the broad-band shape of the power spectra. In each panel, the thick solid curves show the  $\Delta\chi^2$  values for the projected cross-power spectrum ( $C_{sp}(k)$ ), the thick dashed curves are for the angular cross-power spectrum ( $C_{sp}(l)$ ), and the thin horizontal line is for the projected auto-correlation of the spectroscopic sample ( $C_{ss}(k)$ ). For comparison, we also show the result when the BAO feature is extracted from the three-dimensional power spectrum analysis ( $P_{3D}(k)$ ), which is estimated using Eq. (18). The number density of the spectroscopic sample is fixed to  $(20\Delta z) \text{ deg}^{-2}$ , and the *total* number density of the photometric sample is assumed to  $\bar{n}_p^{\text{tot}} = 50 \text{ arcmin}^{-2}$ . Results are shown for three mean redshift,  $\bar{z}_s = 2.5$  (*top panels*), 1.4 (*middle*), and 0.8 (*bottom*). *Left:*  $\Delta\chi^2$  is calculated under the conditions that the photo- $z$  accuracy is fixed to  $\lambda_z = 0.1$  and the redshift bin width is varied from  $\Delta r = 0.1$  to 1.5 Gpc/h. *Right:*  $\Delta\chi^2$  is calculated with a fixed redshift bin width  $\Delta r = 1.5 \text{ Gpc/h}$ , but with varying photo- $z$  accuracies.

(Schneider et al. 2010; Paris et al. 2012) for a spectroscopic sample, as shown in Fig. 2. For a photometric sample, we assume a hypothetical imaging sample which has a full overlap with the spectroscopic survey region; we assume the total area of  $10,000 \text{ deg}^2$  for the overlapping area. We consider six redshift bins with the mean redshifts ranging from 0.7 to 2.9. The projected number density in each bin is estimated using the redshift distribution in Fig. 2. We use the bias parameters of the quasars in each redshift bin based on the measurement by Ross et al. (2009). For the photometric sample, we again assume the total number density of  $\bar{n}_p^{\text{tot}} = 50 \text{ arcmin}^{-2}$ , and compute the number density in each redshift bin taking into account the photo- $z$  error (see Sec. 2.1). Table 1 summarizes the set of the survey parameters.

The cross-correlation is measured as a function of the

transverse separation between the pairs of the spectroscopic and photometric objects. The transverse separation, the separation distance between each pair perpendicular to the line-of-sight direction, can be inferred from the observed angular separation on the sky,  $R \propto \Delta\theta$  (see Eq. 2). For this conversion, we need to assume a reference cosmological model to relate the observable  $\Delta\theta$  to the quantity  $R$ . Thus the transverse wavenumber is given as

$$k_{\perp, \text{ref}} = \frac{D_A(z)}{D_{A, \text{ref}}(z)} k_{\perp}. \quad (19)$$

The quantities with “ref” are the quantities from the observable assuming a “reference” cosmological model, and the quantities without the subscript denote the underlying true quantities. Since the reference cosmological model assumed generally differs from the underlying true cosmology,



$\bar{z}_s$	$\Delta z_s$	$b_s$	$\beta(\bar{z}_s)$	$\bar{n}_s$ [deg <sup>-2</sup> ]	$k_{\max}$ [h/Mpc]	area [deg <sup>2</sup> ]	$\lambda_z$	$\bar{n}_p$ [10 <sup>4</sup> deg <sup>-2</sup> ]	$\sigma_{D_A}/D_A$
0.7	0.2	1.52	0.352	3	0.21	10,000	0.01	2.4	0.076
							0.1	2.2	0.095
							0.3	1.7	0.132
							spec auto-correlation		0.191
0.9	0.2	1.70	0.333	3	0.23	10,000	0.01	2.4	0.095
							0.1	2.3	0.095
							0.3	1.7	0.137
							spec auto-correlation		0.237
1.2	0.4	2.01	0.299	3.5	0.25	10,000	0.01	4.0	0.084
							0.1	3.9	0.098
							0.3	3.0	0.141
							spec auto-correlation		0.369
1.6	0.4	2.49	0.252	3.5	0.29	10,000	0.01	2.7	0.080
							0.1	2.7	0.103
							0.3	2.4	0.188
							spec auto-correlation		0.475
2.2	0.8	3.36	0.193	10	0.35	10,000	0.01	2.3	0.068
							0.1	2.6	0.084
							0.3	3.2	0.188
							spec auto-correlation		0.302
2.9	0.6	4.60	0.144	10	0.42	10,000	0.01	0.5	0.075
							0.1	0.6	0.133
							0.3	1.4	0.536
							spec auto-correlation		0.306

**Table 1.** A summary of survey parameters we consider for the forecast, and the expected fractional errors of determining the angular diameter distance,  $\sigma(D_A)/D_A$ , including marginalization over the other parameters. Here we consider the SDSS/BOSS spectroscopic quasar catalogue for the spectroscopic sample, and the Subaru HSC- or Euclid-type galaxy sample for the photometric sample.  $\bar{z}_s$  and  $\Delta z_s$  are the mean redshift and the redshift width for each redshift bin of the spectroscopic sample taken in the hypothetical cross-correlation analysis.  $b_s$ ,  $\beta$  and  $\bar{n}_s$  are the linear bias, the linear redshift-space distortion and the number density in each redshift bin (see text for the details).  $k_{\max}$  is the maximum wavelength used for the Fisher matrix analysis. For each redshift bin, we cross-correlate the spectroscopic sample with the photometric galaxies based on their photo- $z$ 's assuming the photo- $z$  errors of  $\lambda_z = 0.01, 0.1$  and  $0.3$ , respectively (see Eq. 16).  $n_p$  is the number density of the photometric galaxies in each redshift bin (see Eq. 12). The last column ( $\sigma_{D_A}/D_A$ ) shows the expected error on the angular diameter distance measurement in each bin. For comparison, we also show the expected error when using the three-dimensional auto-power spectrum of the spectroscopic sample (“spec auto-correlation”).

it causes an apparent distortion in the cross-power spectrum – the so-called Alcock-Paczynski test (Alcock & Paczynski 1979; Eisenstein & Hu 1998; Seo & Eisenstein 2003). Thus the observed cross-power spectrum is given as

$$C_{sp}^{\text{obs}}(k_{\perp, \text{ref}}; z) = \frac{D_{A, \text{ref}}(z)^2}{D_A(z)^2} C_{sp}(k; z) + \mathcal{P}_s(z), \quad (20)$$

where  $\mathcal{P}_s(z)$  is the residual shot noise<sup>15</sup>. Since we consider a wide redshift bin for the projection of the spectroscopic sample, we can safely ignore the redshift-space distortion effect.

To make a parameter forecast, we include the following set of parameters:

$$\theta = \{\Omega_m, \Omega_m h^2, \Omega_b h^2, A_s, n_s, \alpha_s, D_A(z_i), \mathcal{A}(z_i), \mathcal{P}_s(z_i)\}, \quad (21)$$

<sup>15</sup> Suppose that the spectroscopic and photometric samples reside in their host haloes, which have the number densities of  $n_{h1}$  and  $n_{h2}$ , and assume that some fractions of the two samples have the common host haloes which have the number density of  $n_c$ . In this case, the shot noise for the cross-power spectrum is found to be proportional to  $n_c/(n_{h1}n_{h2})$ .

where  $\Omega_m h^2$  and  $\Omega_b h^2$  are the matter and baryon density parameters today,  $A_s$  is the amplitude of the primordial curvature perturbation at  $k_{\text{piv}} = 0.005$  Mpc, and  $n_s$  and  $\alpha_s$  are the tilt and running of the primordial power spectrum (Komatsu et al. 2011). The parameter  $D_A(z_i)$  is the angular diameter distance to the  $i$ -th redshift bin which is treated as an independent parameter from other cosmological parameters (see e.g., Seo & Eisenstein 2003; Ellis et al. 2012, for details). We also include the normalization parameter  $\mathcal{A}(z_i)$  which models an uncertainty in the normalization of the cross-power spectrum in each redshift bin due to unknown bias uncertainties for both the photometric ( $b_p$ ) and spectroscopic ( $b_s$ ) samples. As discussed above, the marginalization over  $\mathcal{A}(z_i)$  also takes account of photo- $z$  uncertainties.  $\mathcal{P}_s(z_i)$  is a nuisance parameter to model the residual shot noise parameter. In addition to these parameters, we include the optical depth and angular diameter distance to the last scattering surface,  $\tau$  and  $D_{A, \text{CMB}}$ , respectively to the Fisher matrix to describe the CMB prior.

The full Fisher matrix can be expressed by a simple sum of two Fisher matrices,  $\mathbf{F} = \mathbf{F}^{\text{CMB}} + \mathbf{F}^{\text{CC}}$ , where  $\mathbf{F}^{\text{CC}}$  denotes the Fisher matrix from the cross-correlation mea-

surement:

$$F_{\alpha\beta}^{\text{CC}} = \sum_i^{N_{\text{bin}}} \sum_{k_n=k_f}^{k_{\text{max}}} \frac{\partial C_{sp}(k_n, z_i)}{\partial \theta_\alpha} \text{Cov}^{-1} \frac{\partial C_{sp}(k_n, z_i)}{\partial \theta_\beta}, \quad (22)$$

where Cov is the covariance matrix given by Eq. (14). The maximum wavenumber  $k_{\text{max}}$  is set to the maximum scale up to which the nonlinear matter power spectrum at the mean redshift is expected to be accurate to within 1% level as in Fig. 4.

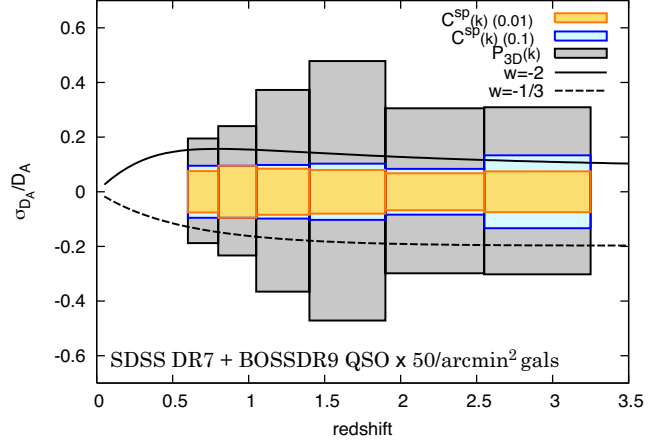
For comparison, we also show a forecast for using the three-dimensional power spectrum of the spectroscopic sample to estimate the cosmological distances,  $H(z_i)$  and  $D_A(z_i)$ . We follow the methods in Seo & Eisenstein (2007) (also see Ellis et al. 2012). We model the redshift-space distortion (RSD; Kaiser 1987) and its nonlinear effects (Eisenstein et al. 2007) for the three-dimensional power spectrum with additional parameters;  $\beta_i = d \ln D(z_i) / d \ln a / b_s$  and  $H(z_i)$ . The fiducial value of  $\beta$  is listed in Table 1. However the results are not sensitive to the details, because the power spectrum information at relevant wavenumber bins is limited by the shot noise contamination for the sparse spectroscopic sample we are interested in.

Table 1 and Fig. 6 show an expected accuracy of the angular diameter distance measurement in each redshift bin via the cross-correlation method. The cross-correlation method allows an improvement in the geometrical test compared to the three-dimensional auto-power spectrum analysis, by reducing the shot noise contamination. For the SDSS/BOSS spectroscopic quasar catalogues, the cross-correlation method enables the fractional accuracy better than 10% in each redshift bin, if the photometric galaxy survey has a full overlap with the SDSS/BOSS footprints and we can select adequate galaxy samples whose photo- $z$  errors are better than  $\lambda_z = 0.1 - 0.2$  (see Eq. 16). Also, an advantage of this method is to determine the angular diameter distance up to a high redshift of  $z \simeq 3$ , where the cosmic expansion is well in the decelerating expansion phase.

## 5 SUMMARY

In this paper, we have studied how the cross-correlation between the spectroscopic and photometric samples can be used for the two-dimensional BAO experiment. We have shown that, with the aid of the spectroscopic redshift, the cross-correlation preserves the BAO feature in the probed transverse scales, even for the projection over different redshifts such as  $\Delta z \simeq 1$ , while the angular (cross)-correlation suffers from a smearing of the BAO feature due to unavoidable photo- $z$  errors that cause a mixing of the different physical scales in a particular angular scale (see Fig. 3). There are several notable advantages of this method. First, the cross-correlation allows a significant reduce in the shot noise contamination to the measurement. Second, any statistical or systematic (catastrophic) photo- $z$  errors affect only the overall normalization of the cross-correlation function, and do not change the shape of the power spectrum.

The cross-correlation method can be useful, if the spectroscopic sample has a wide coverage of redshift, but does not have a sufficiently high number density for the BAO experiment via the auto-correlation analysis. As a specific



**Figure 6.** Expected accuracies on the angular diameter distance measurements with the cross-correlation BAO analysis as in Table 1. We show the results for the photo- $z$  accuracies of both  $\lambda_z = 0.01$  and  $0.1$ , denoted as  $C_{sp}(k)(0.01)$  or  $C_{sp}(k)(0.1)$ , respectively. The outermost boxes show the expected accuracies when the auto-correlation power spectrum of the spectroscopic quasar catalogue is used. The spectroscopic sample is divided into 6 sub-samples with their spectroscopic redshifts. The solid and dashed curves show the changes in the angular diameter distance when the dark energy equation of state ( $w$ ) is changed to  $-2$  and  $-1/3$  from  $w = -1$  (the cosmological constant).

example, we have considered the SDSS and BOSS spectroscopic quasar sample to estimate the feasibility of the cross-correlation method, motivated by the fact that wide-area imaging surveys such as the Subaru HSC Survey and Euclid are being planned, which overlap with the SDSS survey footprints (see Fig. 2). Here the SDSS quasar sample has a wide redshift coverage of  $0 < z \lesssim 4$  and wide area coverage of about  $10000 \text{ deg}^2$ , but has too small number density of  $\sim 10^2 \text{ deg}^{-2}$  per unit redshift interval to implement the BAO experiment via the auto-correlation analysis. On the other hand, the planned imaging surveys likely provide a much denser sampling of galaxies such as  $10^3 - 10^5 \text{ deg}^{-2}$  over the redshift range. We have shown that the cross-correlation allows a more accurate BAO measurement over  $0.7 < z \lesssim 3$  than in the auto-correlation of the spectroscopic sample or the angular power spectrum of the photometric galaxies (see Figs. 5 and 6 and Table 1), if the photometric redshift is reasonably good, 10 – 20% level in the fractional accuracy, in order not to have a severe dilution in the measured cross-correlation. As shown in Fig. 6, the better photo- $z$  accuracy of  $\sigma_z / (1 + \bar{z}_s) = 1\%$  does not improve constraints on  $D_A$  significantly compared to the 10% photo- $z$  accuracy. Hence the 10 – 20% of the photo- $z$  accuracy is sufficient for the cross-correlation BAO study, which can easily be achieved for the current and upcoming multi-band photometric galaxy surveys. Large-scale structure is more in the linear regime at higher redshift due to less evolving nonlinearities, and therefore we can use the shape of the cross-power spectrum, including the BAO feature, up to the higher maximum wavenumber, thereby yielding a more constraining power of the angular diameter distance.

The expected accuracy of the angular distance measurement in Fig. 6 is from both the BAO feature and the broad shape of the power spectrum. The projected cross-

correlation allows us to measure the shape of the *three-dimensional* power spectrum, although the overall normalization is affected by photo- $z$  errors. Hence, the method can also be used to constrain the tilt and running index of the primordial power spectrum. Also, as an ultimate possibility, the cross-correlation method may enable to use the observed radius of dark matter haloes in the projected distance. If we have a good knowledge on the virial radius of dark matter haloes as well as have a good estimator of halo masses, observed the virial radius can be used to infer the angular diameter distance. This is relevant for cluster-shear weak lensing, which probes the halo and dark matter cross-correlation (Oguri & Takada 2011). Given that the clusters have follow-up spectroscopic redshifts, we can expect a high-precision measurement of the halo-matter cross correlation at small scales down to a few Mpc, which correspond to the virial radii of massive haloes. Thus the virial radius may serve as another standard ruler that can lead to even higher-precision measurements of the angular diameter distances. This is an interesting possibility and may worth exploring further.

We note that the cross-correlation technique developed in this paper can also be used to constrain the primordial non-Gaussianity via measurements of the largest-scale cross-power spectra (Dalal et al. 2008; Taruya et al. 2008; McDonald 2008; Slosar et al. 2008). Again, by measuring the cross-correlation as a function of the transverse comoving separation, we can avoid the the smearing effect due to the projection which reduces the enhanced power at the largest scales of  $k \lesssim 0.01h/\text{Mpc}$ .

Spectroscopic observations of quasars, or more generally bright, rare galaxies, are relatively inexpensive in terms of the observation time needed for a given telescope. Such objects are also very interesting subjects for astronomical studies. The method developed in this paper can add a cosmological science case when combined with wide-area imaging surveys that have an overlap with the spectroscopic survey. The method is useful when designing joint spectroscopic and photometric surveys including a science case of the two-dimensional BAO analysis.

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